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Composition of Angular Momentum and Its Applications in Human Being

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***Abstract*: This paper explores the many uses of angular momentum regulation and its role in the synthesis of co—ordinate motion generated with physical based characters. We find that students have many common difficulties related to basic of angular momentum and its application in human being. Angular momentum is the force which a moving body, following a curved path, has because of its mass and motion. Neutral movement of humans reveals a high degree of co-ordination in which the entire body moves in concert to perform a given task. We describe the development and implementation of a research—based learning tool, to reduce to understand angular momentum. There have been many investigations of student's difficulties in learning quantum mechanics (QM). Scientists have collectively performed the most through study of the role of angular momentum in human movement to date. They postulate that whole body angular momentum may be regulated directly by the central nervous system. Here, we have seen that it is possible to use momentum to produce coordinated motion such as balance, stepping and walking through a multi-objective framework. Several key observations will be made in this paper. The concepts described in this paper highlight the importance of angular momentum control over whole body co-ordinate behaviors in all human activities.**

***Keywords*: centre of mass (cm), centre of pressure (cp), linear momentum, spin, Clehsch Gordon.**

I. INTRODUCTION

Angular momentum can be arranged as a vector operator of a special kind whose three components do not commute with each other. In classical mechanics the orbital angular momentum of a particle relative to the centre of rotation is a vector quantity L. It is given by the following equation.

$$\vec{L} = \vec{r} \text{ x } \vec{p}$$

Where *r* is the position vector and *p* is the linear momentum of the particle. In case of many problems the angular momentum is a constant of motion. Therefore, it plays a very important role in quantum mechanics. The orbital angular momentum of a body is associated with its rotation about a certain axis. In classical mechanics the orbital angular momentum of a particle about a point is defined as the momentum of linear momentum of the body about the axis of rotation. The components of the angular momentum, $\vec{L}$, about *x*,*y* and *z* axis using vector analysis can be represented as follows[1].

$$\vec{L} = iL_x + jL_y + kL_z$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{p} = ip_x + jp_y = kp_z$$

We can get components

$$L_x = yP_z - zP_y$$

$$L_y = zP_z - xP_z$$

$$L_z = xP_y - yP_x$$

Communitation rules for orbital angular momentum as follows

$$[L_x, L_y] = ihL_z$$

$$[L_y, L_z] = ihL_x$$

$$[L_x, L_z] = ihL_y$$

The given values of L_x is given by the equation

$$L\psi = c'\psi$$

And the solution of above equation is given by

$$\psi = f(r, \theta)\, e^{i'c'\phi/h}$$

The total angular momentum operators $\vec{J} = \vec{L} + \vec{S}$ and communication relations obeyed by components of generalized momentum operator is

$$[J_x J_y] = ihJ_z$$

$$[J_y J_z] = ihJ_x$$

$$[J_z J_x] = ihJ_y$$

Composition of Angular momentum [2]:

In the case of hyperfine structure of the hydrogen atom, we must know the internal states of a system composed of two particles, the electron and the proton each with a spin of one half. We found that the four possible spin states of such a system could be put together into two groups - a group with one energy that looked to the external world like a spin, one particle, and one remaining state that behaved like a particle of zero spin. That is putting together two spin one half particles we can form a system whose “total spin” is one, or zero. In this paper, we want to discuss in more general terms the spin states of a system which is made up of two particles of arbitrary spin. It is another important problem about angular momentum in quantum mechanical systems[3].

Let’s first rewrite the hydrogen atom in a form that will be easier to extend to the more general case. We began with two particles which we will now call particle *a* (the electron) and particle *b* (the proton). Particle *a* had the spin $j_a\ (=\tfrac{1}{2})$, and its *z*—component of angular momentum m_a could have one of several values (actually 2, namely $m_a + \tfrac{1}{2}$ or $m_a = -\tfrac{1}{2}$). Similarly, the spin state of particle *b* is described by its spin j_b and its *z* - component of angular momentum m_b. various combinations of the spin states of the two particles could be formed. For instance, we could have particle *a* with $m_a = \tfrac{1}{2}$ and particles *w*ith $m_b = -\tfrac{1}{2}$, to make a state $[a, +\tfrac{1}{2}; b, -\tfrac{1}{2}]$. In general, the combined states formed a system whose ‘system spin..” or “total spin,” or “total angular momentum” J could be 1, or 0. And the system could have a *z*—component of angular momentum *M*, which was +1, 0 or —1 when J = 1, or 0 when J = 0.We want now to generalize this result to states made up of two objects *a* and *b* of arbitrary spin j_a and j_b. We start by considering an example for which $j_a = \tfrac{1}{2}$

Composition of Angular Momentum for two spins $\tfrac{1}{2}$ particles $\left(j_a = \tfrac{1}{2},\ j_b = \tfrac{1}{2}\right)$

$$|J=1, M=+1\rangle = |a, +\tfrac{1}{2}; b+\tfrac{1}{2}\rangle$$

$$|J=1, M=0\rangle = \frac{1}{\sqrt{2}}\left\{|a, +\tfrac{1}{2}; b, -\tfrac{1}{2}\rangle + |a, -\tfrac{1}{2}; b, +\tfrac{1}{2}\rangle\right\}$$

$$|J=1, M=-1\rangle = |a, -\tfrac{1}{2}; b, -\tfrac{1}{2}\rangle$$

$$|J=0, M=0\rangle = \frac{1}{\sqrt{2}}\{|a, +\tfrac{1}{2}; b, -\tfrac{1}{2}\rangle - |a, -\tfrac{1}{2}; b, +\tfrac{1}{2}\rangle\}$$

And $j_b = 1$, namely, the deuterium atom in which particle a is an electron (e) and particle b is the nucleus - a deuteron (d). [4 to 9]We have then that $j_a = j_e = \tfrac{1}{2}$. the deuteron is formed of one proton and one neutron in a state whose total spin is one, so $j_b = j_d = 1$. we want to discuss the hyperfine states of deuterium - just the way we did for hydrogen. Since the deuteron has three possible states $m_b = m_d = \_\ 1, 0, -1$, and the electron has two, $m_a = m_e = +\tfrac{1}{2}, -\tfrac{1}{2}$, there are six possible states as follows (using the notation $|e, m_e; d, m_d\rangle$):

$$|e, +\tfrac{1}{2}; d, +1\rangle$$

$$|e, +\tfrac{1}{2}; d, 0\rangle ; |e, -\tfrac{1}{2}; \mathrm{d}+1\rangle ,$$

$$|e, +\tfrac{1}{2}; d, -1\rangle ; |e, -\tfrac{1}{2}; d, 0\rangle ,$$

$$|e, -\tfrac{1}{2}; d, -1\rangle$$

If the new system is just rotated about the z—axis by the angleϕ, then the state $|e, m_e; d, m_d\rangle$ gets multiplied by

$$e^{im_e\phi}e^{im_d\phi} = e^{i(m_e+m_d)\phi}$$

(The state may be thought of as the product $|e, m_e\rangle|d, m_d\rangle$, and each state vector contributes independently its own exponential factor).

$$M = m_e + m_d$$

The z—component of the total angular momentum is the sum of the z—components of angular momentum of the parts.[10,11,12]

In the list of, therefore, the state in the top line has $M = +\tfrac{3}{4}$, the two in the second line have $M = +\tfrac{1}{2}$, the next two have $M = -\tfrac{1}{2}$, and the last state has $M = -\tfrac{3}{2}$. We see immediately one possibility for the spin J of the combined state (the total angular momentum) must be $\tfrac{3}{2}$, and this will require four states with $M = +\tfrac{3}{2} + \tfrac{1}{2}, -\tfrac{1}{2}$, and - $\tfrac{3}{2}$.

There is only one candidate for $M = \tfrac{3}{2}$, so we know already that

$$|J=\tfrac{3}{2}, M=+\tfrac{3}{2}\rangle = |e, +\tfrac{1}{2}; d, +1\rangle$$

But what is the state $|J=\tfrac{3}{2}, M=\tfrac{1}{2}\rangle$? We have two candidates and in fact, any linear combination of them would also have $M = \tfrac{1}{2}$. So, in general, we must expect to find that

$$|J=\tfrac{3}{2}, M=+\tfrac{1}{2}\rangle = \alpha|e, +\tfrac{1}{2}; d, 0\rangle + \beta|e, -\tfrac{1}{2}; d, +1\rangle ,$$

Whereα and β are two numbers. They are called the Clebsch—Gordon coefficients.

Addition of Angular Momenta and Clehsch Gordon [13, 14]:

Many times we have to deal with systems in which the total angular momentum consists of two or more parts which are, to some approximation at least, are independent of each other. The examples are the particles with spin (in the non-relativistic limit) systems containing two or more particles such as many electron atoms the scattering and radiation processes etc. The problem is to find how the total angular momentum is related to its component parts.

The old quantum theory was facing the problem that how to combine the angular momenta associated with two parts of a system to form the angular momentum of the whole system The vector model solved this problem by applying an additional rule which states that the magnitude of the sum of two angular momentum vectors can have any value ranging from the sum of their magnitudes (parallel case) to the difference of their magnitudes (antiparallel case) by integral steps. The vector model also states that the sum of the z components of the angular momenta equals that of their resultant. Both these rules are true in case of quantum mechanics as well.

Applications in human being [15]:

We study about angular momentum control in biomechanics, control in robotics and in animation. We have studied walking motion in depth and report observing surprisingly small angular momentum valves in straight line walking which lead to very small angular excursions over entire cycles of normal subjects, walking. Based on such observations, they propose that walking is regulated to have zero spin (zs) angular momentum about the centre of mass (CM). For walking and other "zs" behaviors, their hypothesis is that both angular momentum and its time derivative are regulated to remain close to zero. Several of their various findings support this hypothesis. Along with data analysis and models of human subjects, these researchers have also spelled out the value of regulating angular momentum in control for humanoid robots and they have implemented and described a handful of simulations. A human model was constructed in order to calculate physical quantities such as cm position and the angular momentum. The model and co-ordinate system are used in this study. The model comprises 16 rigid body segments, feet, tibias, femurs, hands, forearms, arms, pelvis – abdomen, chest, neck and head. The feet and hands are modeled as rectangular boxes.

By simple inspection, we can see that the derivative of the linear momentum is the same as the mass-scaled CM acceleration. Note this coupling implies that control over the CM (and its time derivatives) as has been seen in most previous control approaches for locomotion across disciplines is equivalent to control of linear momentum (and its derivatives), assuming mass is constant. We can also see that angular (spin) momentum change completely describes the relationship between the position of the center of mass, c, and the aggregate GEF, *f*, applied at point *p*. Finally, together, along with *f*, these two momenta rates can be integrated to yield the complete rigid motion of the character about its CM.[16].

For balance control to produce stepping behaviors produced by specifying trajectories for the CM and the swing foot. The desired motion of theswing foot is translated into a joint-angle reference trajectory and in doing sowe can use the same solver implemented for the balance paper. However, instead of guiding the CP, we specify angular momentum directly similar to the scheme described by Kajita et al. in their resolved momentum control paper. Specifically, we enforce $H_z = 0$ while the other spin momentum terms are uncontrolled. The effect of this controller is to restrict twist rotation, thereby maintaining a consistent facing direction while allowing rotation in roll and pitch.

II. CONCLUSION

Angular momentum is a conserved physical quantity for isolated systems, where no external moments act about a body's center of mass (cm). However, in the case of legged locomotion, where the body interacts with the environment, there is no a priori reason for this relationship. The body's cm location is estimated using the human model and joint position data from the motion capture measurements. Whole body angular momentum is calculated using human model and kinetic gait data. Angular momentum L is calculated as the sum of individual segment angular momenta about the body's cm. An understanding of angular momentum behaviors in human walking and other movement tasks may have important implication for several fields of study.

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